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**UNSTEADY HYBRID VORTEX TECHNIQUE FOR TRANSONIC
VORTEX FLOWS AND FLUTTER APPLICATIONS**

By

Osama A. Kandil, Principal Investigator

Progress Report

For the period January 1, 1987 - July 31, 1987

Prepared for the

National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia 23665

Under

Research Grant NAG-1-648

Dr. Samuel R. Bland, Technical Monitor
LAD-Unsteady Aerodynamics Branch

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Submitted by the
Old Dominion University Research Foundation
P.O. Box 6369
Norfolk, Virginia 23508

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By

Osama A. Kandil*

SUMMARY

This report covers the progress of research work performed under this grant in the period of January 1, 1987 to July 31, 1987. During this period of seven months, the following papers which report on the accomplishments and results have been presented and published (copies of some of these papers are attached to this report):

I. Integral Equation Solution of the Full-Potential Equation for Steady and Unsteady Transonic Flows:

- A. "Integral Equation Solution For Transonic And Subsonic Aerodynamics." Kandil, O. A. and Hu, H., presented at the Third GAMM-Seminar on Panel Methods in Mechanics, Kiel, West Germany, January 16-18, 1987. Published in "Notes on Numerical Fluid Mechanics". B.
- B. "Full-Potential Integral Solution for Transonic Flows With And Without Embedded Euler Domains," Kandil, O.A. and Hu, H., AIAA 87-1461, Honolulu, Hawaii, June 8-10, 1987. Submitted for publication to AIAA Journal.
- C. "Transonic Airfoil Computation Using the Integral Equation With and Without Embedded Euler Domains," Kandil, O.A. and Hu, H., 9th International Conference on "Boundary Element Methods in Engineering, University of Stuttgart, Stuttgart, West Germany,

*Professor, Department of Mechanical Engineering and Mechanics, Old Dominion University, Norfolk, Virginia 23508

- E. "Influence of Numerical Dissipation on Computational Euler Equations for Vortex-Dominated Flows," Kandil, O.A. and Chuang, A.H., AIAA Journal, Vol. 25, No.9, September 1987.

III. The following abstracts have been submitted for presentations:

- A. "Unsteady Vortex-Dominated Flow Around Maneuvering Wings Over a Wide Range of Mach Numbers," Kandil, O.A. and Chuang, A.H., submitted for presentation at AIAA 26 ASM, Reno, Nevada, January 11-14, 1988.
- B. "Unsteady Transonic Airfoil Computation Using the Integral Equation Solution," Kandil, O.A. and Hu, H., AIAA 27 SDM Conference, Williamsburg, Virginia, April 19-21, 1988.

IV. The following talks will be given at the ICASE, Nasa Langley:

- A. "Full-Potential Integral Solutions for Steady and Unsteady Transonic Airfoil computations With and Without Embedded Euler Domains," Hu, H. and Kandil, O.A., August 18, 1987.
- B. "Steady and Unsteady Euler Computations of Various Flow Regimes Past Delta Wings," Chuang, A.H. and Kandil, O.A., August 27, 1987.

V. The Principal Investigator has chaired and organized the following sessions and conferences of the AIAA:

General Chairman and organizer, Fluid Dynamics Sessions (29 Sessions), AIAA 25th Aerospace Sciences Meeting, Reno, Nevada, January 12-15, 1987.

- B. Chairman and organizer, Incompressible Navier Stokes II, AIAA 8th Computational Fluid Dynamics Conference, Honolulu, Hawaii, June 9-11, 1987.
- C. Chairman and organizer, Leading-Edge Vortex Flows, AIAA 9th Fluid Dynamics, Plasma Dynamics and Lasers, Honolulu, Hawaii, June 8-11, 1987.
- D. Chairman and organizer, Vortex Flow II, AIAA 25th Aerospace Sciences Meeting, Reno, Nevada, January 12-15, 1987.

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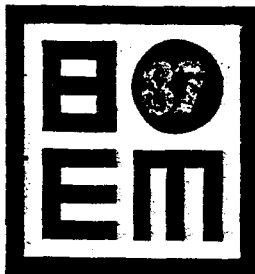
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TRANSONIC AIRFOIL COMPUTATION USING
THE INTEGRAL EQUATION WITH AND
WITHOUT EMBEDDED EULER DOMAINS

OSAMA A. KANDIL AND HONG HU

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INTRODUCTION

Computation of the transonic flow around airfoils and wings using finite-difference and finite-volume methods requires fine grids and large computational domains around the source of disturbance. The outer boundary of the computational domain is usually placed at several chord lengths away from the inner boundary. Moreover, special treatment is required at the outer boundary to approximately satisfy the farfield boundary conditions. Using different levels of approximations, inviscid computational schemes have been developed based on the Transonic Small Perturbation (TSP) equation (Murman and Cole¹; Edwards, Bland and Siedel²), Full Potential (FP) equation (Steger and Lomax³; Garabedian and Korn⁴; Jameson⁵) and Euler equation (Jameson⁶). These schemes require large capacity of computer memory to handle the large number of grid points and the associated flow variables. They also require large CPU time to obtain converged solutions due to the large number of iteration cycles — usually it is of order a thousand.

The potential equations can be used for flows with weak shocks since the entropy increase and vorticity production across these shocks are small. For strong shocks, the irrotational and isentropic flow assumptions are invalid and one cannot use the potential flow equations across or behind the shocks. For these flows, one has to correct the potential equations in order to include the entropy jump across the shock wave. Methods of this type have been developed by Hafez and Lovell⁷, Fuglsang and Williams⁸ and Whitlow, Hafez and Osher⁹. Alternatively, one has to use the computationally

more expensive Euler equations.

The Integral Equation Formulation (IEF) using the TSP (Piers and Sloof¹⁰; Tseng and Morino¹¹) or the FP (Kandil and Yates¹²; Oskam¹³; Erickson and Strande¹⁴; Sinclair¹⁵; Kandil and Hu¹⁶) equations represents an alternative to the finite difference and finite volume methods to treat transonic flows with weak shocks. With the IEF, the farfield boundary conditions are automatically satisfied and only a small computational domain is needed around the source of disturbance. Moreover, the accuracy of the method depends on the evaluation of integrals rather than derivatives and hence coarse grids can be used within the small computational domain. Because of these obvious advantages of the methods which are based on the IEF, it is highly desirable to fully develop these methods and extend them to treat transonic flows over a wide range of Mach numbers.

In this paper, we present a transonic integral equation method which is based on the full potential equation, and couple the method with embedded Euler domains to treat strong shocks. The method is extensively applied to different airfoil sections and Mach numbers. The results are compared with experimental data and other results which were obtained by using finite-difference and finite-volume methods with TSP, FP and Euler equations.

FORMULATION

Full Potential Equation

For transonic flows with weak shocks, the dimensionless governing equations of the two-dimensional, steady, potential flow are given by

$$\Phi_{xx} + \Phi_{yy} = G \quad (1)$$

$$G = \frac{-1}{\rho} (\rho_x \Phi_x + \rho_y \Phi_y) \quad (2)$$

$$\rho = [1 + \frac{\gamma-1}{2} M_\infty^2 (1 - \Phi_x^2 - \Phi_y^2)]^{1/\gamma-1} \quad (3)$$

where Φ is the total velocity potential, ρ the density, M_∞ the freestream Mach number, γ the ratio of specific heats and the subscripts x and y refer to the derivatives. It should be noted that G represents the total compressibility in the flow. The characteristic parameters are the freestream velocity (U_∞) and density (ρ_∞) and the airfoil chord length (l).

The boundary conditions on the airfoil g , away from g and at the trailing edge TE are given by

$$\nabla\Phi \cdot \hat{n} = 0 \quad \text{on } g(x, y) = 0 \quad (4)$$

$$\nabla\Phi \rightarrow \bar{e}_\infty \quad \text{away from } g \quad (5)$$

$$\Delta C_p|_{TE} = 0 \quad (6)$$

where $\hat{n} = \nabla g / |\nabla g|$, \bar{e}_∞ is a unit vector parallel to \bar{U}_∞ and ΔC_p is the pressure jump. The pressure coefficient is given by

$$C_p = \frac{2}{\gamma M_\infty^2} \{ [1 + \frac{\gamma-1}{2} M_\infty^2 (1 - \Phi_x^2 - \Phi_y^2)]^{\gamma/\gamma-1} - 1 \} \quad (7)$$

The formal solution of Eq. (1) in terms of the velocity field $\nabla\Phi$ with explicit contribution of the shock surface S is obtained as

$$\begin{aligned} \nabla\Phi(x, y) = & \bar{e}_\infty + \frac{1}{2\pi} \oint_g q_g(s) \frac{(x-\xi) \bar{i} + (y-\eta) \bar{j}}{(x-\xi)^2 + (y-\eta)^2} ds \\ & + \frac{1}{2\pi} \oint_g \gamma_g(s) \frac{(y-\eta) \bar{i} - (x-\xi) \bar{j}}{(x-\xi)^2 + (y-\eta)^2} ds \\ & + \frac{1}{2\pi} \iint G(\xi, \eta) \frac{(x-\xi) \bar{i} + (y-\eta) \bar{j}}{(x-\xi)^2 + (y-\eta)^2} d\xi d\eta \\ & + \frac{1}{2\pi} \oint_S q_S(s) \frac{(x-\xi) \bar{i} + (y-\eta) \bar{j}}{(x-\xi)^2 + (y-\eta)^2} ds \end{aligned} \quad (8)$$

where q_g and γ_g are the airfoil surface distributions of sources and vorticity; respectively, and q_S is the source strength of the shock surface. In the shock-capturing scheme, the last integral term in Eq. (8) is dropped, since this term is included in the third integral term of the equation. In the shock capturing-shock fitting scheme, the last integral term corresponding to the shock surface is retained when shock fitting is used. For shock fitting, the following equations are used to determine the shock strength q_S , properties behind

the shock ρ_2 , V_{2n} , V_{2t} and the orientation of the segments forming the shock surface β :

$$q_S = - (V_{1n} - V_{2n}) = - \frac{2 V_{1n}}{\gamma+1} \left(1 - \frac{1}{M_{1n}^2}\right), \quad M_{1n} > 1 \quad (9)$$

$$V_{2n} = \frac{(\gamma-1) M_{1n}^2 + 2}{(\gamma+1) M_{1n}^2} V_{1n} \quad (10)$$

$$V_{2t} = V_{1t} \quad (11)$$

$$\rho_2 = \frac{(\gamma+1) M_{1n}^2}{(\gamma-1) M_{1n}^2 + 2} \rho_1 \quad (12)$$

$$\beta = \sin^{-1} \left[\frac{1.2 \sin\theta \sin\theta}{\cos(\beta-\theta)} + \frac{1}{M_1^2} \right]^{1/2} \quad (13)$$

$$M_1 = M_\infty |\nabla\Phi|_1 / \rho_1^{\frac{\gamma-1}{2}}, \quad M_{1n} = M_\infty \nabla\Phi_1 \cdot \hat{n}_S / \rho_1^{\frac{\gamma-1}{2}} \quad (14)$$

where the subscripts 1 and 2 refer to the conditions ahead and behind the shock, respectively, while n and t refer to the normal and tangential directions to the shock surface S, respectively, and θ is the relative direction of the flow behind the shock to that ahead of the shock.

Euler Equations

For strong shocks, an embedded computational domain is constructed around the shock which has been preliminary found by the Integral solution with shock capturing only. With the boundary and initial conditions found from the Integral solution, the unsteady conservative form of the Euler equations are solved in this limited domain with psuedo time marching. The dimensionless conservative form of these equations is given by

$$\frac{\partial \bar{q}}{\partial t} + \frac{\partial \bar{E}}{\partial x} + \frac{\partial \bar{F}}{\partial y} = 0 \quad (15)$$

where the flow vector field \bar{q} and the flux components \bar{E} and

\bar{F} are given by

$$\bar{q} = [\rho, \rho u, \rho v, \rho e]^t \quad (16)$$

$$\bar{E} = [\rho u, \rho u^2 + p, \rho uv, \rho uh]^t \quad (17)$$

$$\bar{F} = [\rho v, \rho uv, \rho v^2 + p, \rho vh]^t \quad (18)$$

The total energy and enthalpy per unit mass are given by

$$e = \frac{p}{(\gamma+1)\rho} + (u^2 + v^2)/2, \quad h = e + p/\rho \quad (19)$$

Since we are interested in the steady flow solution only, the energy equation [last elements in Eqs. (16)-(18)], which is a differential equation, is replaced by the algebraic steady form which states that the total enthalpy is constant. Hence, the energy equation is replaced by

$$p = (\rho/\gamma) [1/M_\infty^2 + (\gamma-1) (1 - u^2 - v^2)/2] \quad (20)$$

METHOD OF SOLUTION

Shock-Capturing Shock-Fitting (SCSF) Scheme

The basic difference between the incompressible Integral Equation Solution and the transonic Integral Equation Solution are the additional third and fourth integral terms of Eq. (8). In the shock capturing part of the scheme, the fourth integral term is dropped while in the shock fitting part of the scheme this term is taken into account.

The SCSF-scheme is an iterative scheme due to the nonlinearity of the third and fourth integral terms. The iterative scheme is described below:

Neglecting the fourth integral term and setting $G = 0$, a standard panel computation is used to obtain q_g and/or γ_g . The q_g and γ_g are piecewise linear distributions on each surface panel and they are defined in terms of their nodal values. Initial values of G are calculated at the centroids of the field elements by using the linear compressibility relation $G = M_\infty^2 u_x$, where u_x is the x-derivative of the x-component of the velocity. The centroidal value G represents

the G value for the field element. Equation (8) is then used to enforce the surface boundary conditions, Eqs. (5) and (7) to find new q_g and/or γ_g . The density ρ and nonlinear compressibility G are calculated by using Eqs. (3) and (2). A type finite-difference expression is used to calculate ρ_x and ρ_y depending on the type of the centroidal point-subsonic or supersonic. Once the G values are obtained, the surface boundary conditions are satisfied again. The iterative procedure is continued until the shock location is fixed. This is the shock capturing part of the scheme.

Shock panels are then introduced at the shock location, the fourth integral term of Eq. (8) is now taken into account, Eqs. (9) and (13) are used to calculate q_s and β and Eqs. (10)-(12) are used to cross the shock panel. The iterative procedure is continued as before with the exception of dealing with the shock panels as explained. Convergence is achieved once the surface pressure converges. This is the shock fitting part of the scheme.

Integral Equation With Embedded Euler (IEEE) Scheme

In this scheme, the shock capturing part of the SCSF-scheme is used to locate the shock. Once the shock is captured, a fine grid is constructed within a small computational region around the shock where a finite-volume Euler scheme is used. The basic finite-volume equation is obtained by integrating Eq. (15) over x and y to obtain

$$\iint \frac{\partial q}{\partial t} dA + \oint (E dy + F dx) = 0 \quad (21)$$

Equation (21) is then applied to each cell of the embedded grid of the Euler domain. The resulting difference equation is

$$\left(\frac{\partial q}{\partial t}\right)_{ij} \Delta A_{ij} + \sum_{r=1}^4 (E \Delta y_r + F \Delta x_r) = 0 \quad (22)$$

where ΔA is the cell area, r refers to the cell-side number and the integer subscripts i, j refer to the centroidal values. The Euler solver is a central-difference finite-volume method which uses four-stage Runge-Kutta time stepping with explicit second- and fourth-order dissipation terms. The details of this solver are given in reference [17].

The boundary and initial conditions for the Euler domain are obtained from the Integral equation solution which is interpolated on the Euler domain grid. The Euler solver is

then used to capture the shock and calculate the flow vector field q . It should be emphasized here that the downstream boundary condition must be updated while Euler calculations are executed. Fixing the q values of the Euler domain, the IE calculation is used to update the boundary conditions for the Euler domain. The iterative procedure is repeated until convergence is achieved.

COMPUTATIONAL EXAMPLES

In this section, we present applications of the SCSF- and IEEE-schemes to the NACA 0012 and 64A010A. According to the convergence study using different sizes of the IE computational domain, which was presented by the authors (Kandil and Hu¹⁶), a computational domain of 2×1.5 , in the x and y directions, has been used around the airfoil in all the following applications. A rectangular grid of 64×60 has been used for the IE computation. The grid is clustered in the leading edge, the shock region and near the airfoil surface. Since the third integral term of Eq. (8) is computationally expensive, its computation with constant G distribution has been restricted to the nearfield computation. For the farfield computation, this term is replaced by the equivalent lumped source term at its centroid. With sufficient accuracy, it has been computationally determined that the nearfield distance from the centroid is < 0.5 .

Figure 1 shows the results of the SCSF-scheme for NACA 0012, $M_\infty = 0.8$ and $\alpha = 0^\circ$, along with comparisons with the computational results of Garabedian, Korn and Jameson¹⁸, and the experimental data taken from reference 19. The SCSF scheme took 12 iteration cycles of shock capturing (SC) and 13 cycles of shock fitting (SF) to achieve convergence.

Figure 2 shows the results of the IEEE-scheme for the same case along with a comparison with the computational results of Jameson⁶ who also used the finite-volume Euler scheme with four-stage Runge Kutta time stepping. In the present IEEE-scheme, the embedded Euler domain has a size of 0.5×0.6 around the shock region with a grid of 25×30 . This case took 10 iteration cycles of SC, 250 time cycles of Euler iterations to achieve a residual error of 10^{-3} and 5 IE cycles to update the Euler domain boundary conditions.

Figures 3 and 4 show the results of the SCSF- and IEEE-schemes for NACA 64A010A, $M_\infty = 0.796$, $\alpha = 0^\circ$ along with comparisons with the computational results of Edwards, Bland and Seidel² who used the TSP equation, and the experimental data taken from reference 2.* With the SCSF-scheme, the

numbers of SC and SF iteration cycles to achieve convergence are the same as those of the case presented in Fig. 1. With the IEEE-scheme, the embedded Euler domain has a size of 0.7 x 0.6 with a grid size of 35 x 30. This case, Fig. 4, took 10 iteration cycles of SC, 130 time cycles of Euler iterations to achieve a residual error of 10^{-3} and 3 IE cycles to update the Euler domain boundary conditions.

Figures 5 and 6 show the results of the SCSF- and IEEE-schemes for the lifting case of NACA 0012, $M_\infty = 0.75$ and $\alpha = 2^\circ$ along with the computational results of Steger and Lomax³, and the experimental data taken from the same reference. The size of the grids and the number of iteration cycles used to achieve convergence are the same as those of the cases given in Fig. 1 and 2.

Figure 7 shows the results of the IEEE for NACA 0012, $M_\infty = 0.812$ and $\alpha = 0^\circ$ along with the experimental data of reference 18. In Fig. 8, the results of the IEEE for NACA 0012, $M_\infty = 0.82$ and $\alpha = 0^\circ$ are shown along with the three-dimensional solution at the wing root chord of Tseng and Morino¹¹, who use the IE for the TSP, and the experimental results of reference 20. The size of the embedded Euler domain for these cases is 0.8 x 0.8 and the computational grid is 40 x 40.

CONCLUDING REMARKS

Two transonic computational schemes which are based on the Integral Equation Formulation of the full potential equation have been presented. The first scheme is a Shock Capturing-Shock Fitting (SCSF) scheme which uses the full potential equation throughout with the exception of the shock wave where the Rankine-Hugoniot relations are used to cross and fit the shock. The second scheme is an Integral Equation with Embedded Euler (IEEE) scheme which uses the full potential equation with an embedded region where Euler equations are used. The two schemes are applied to several transonic airfoil flows and the results have been compared with numerous computational results and experimental data. The two schemes are nevertheless efficient as compared to the other existing schemes which use finite-difference or finite-volume methods throughout large computational domains with fine grids. The SCSF-scheme is restricted to flows with weak shocks, while the IEEE-scheme can handle strong shocks. Currently, the IEEE scheme is applied to other transonic flows with strong shocks as well as to unsteady pitching oscillations.

ACKNOWLEDGEMENT

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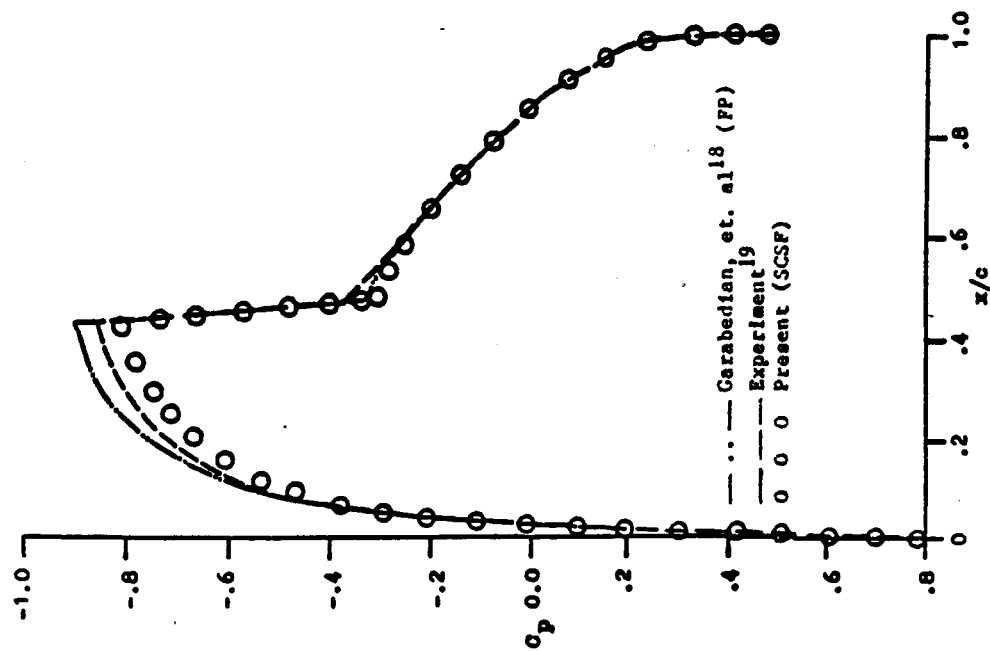


Fig. 1 Integral Eq. Solution with SCSF-Scheme, NACA 0012, $M_\infty = 0.8$, $\alpha = 0^\circ$.

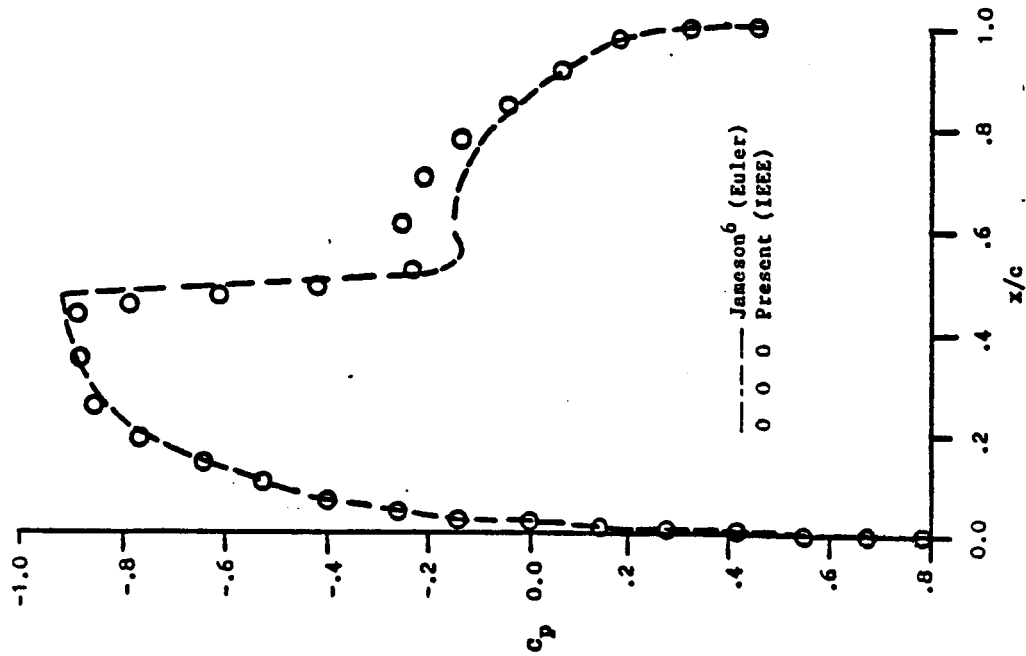


Fig. 2 Integral Eq. with Embedded-Euler Domain Solution, NACA 0012, $M_\infty = 0.8$, $\alpha = 0^\circ$.

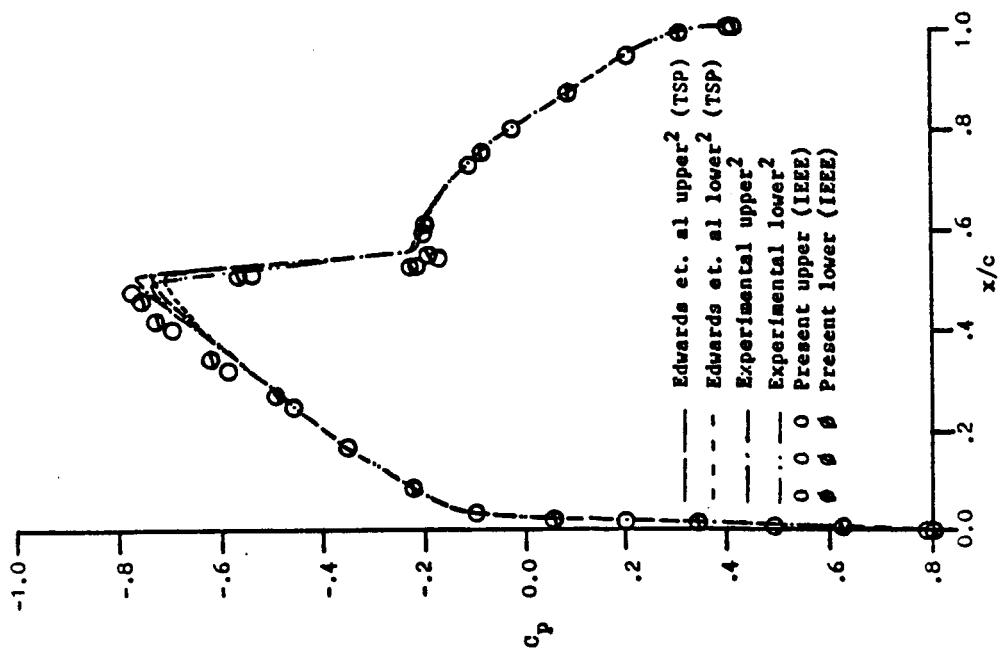


Fig. 3 Integral Eq. Solution with SCSF Scheme, NACA 64A010A, $M_\infty = 0.796$, $\alpha = 0^\circ$.

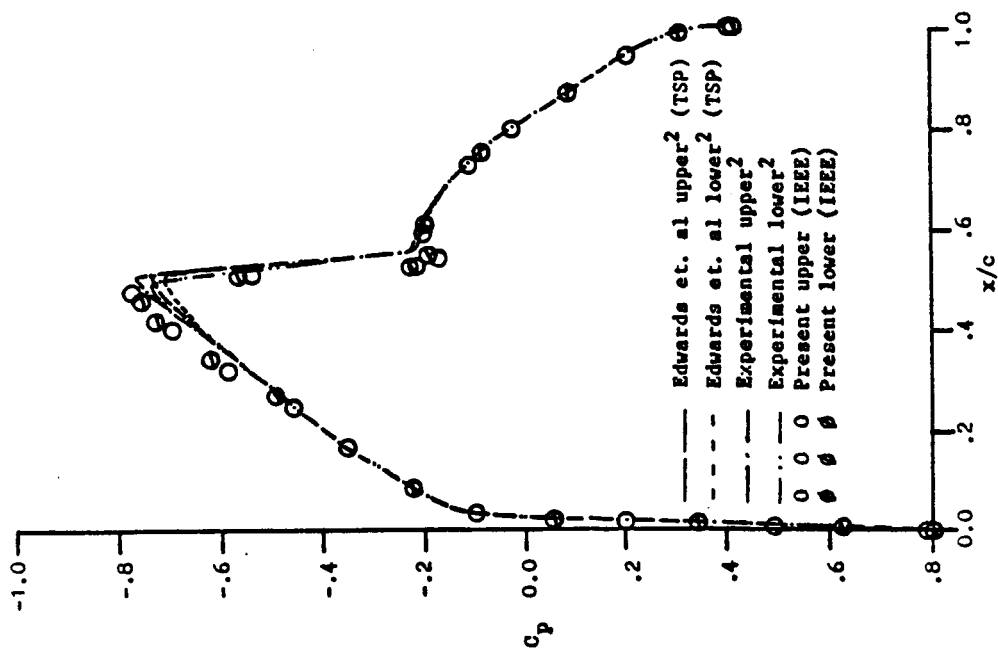


Fig. 4 Integral Eq. with Embedded-Euler Domain Solution, NACA 64A010A, $M_\infty = 0.796$, $\alpha = 0^\circ$.

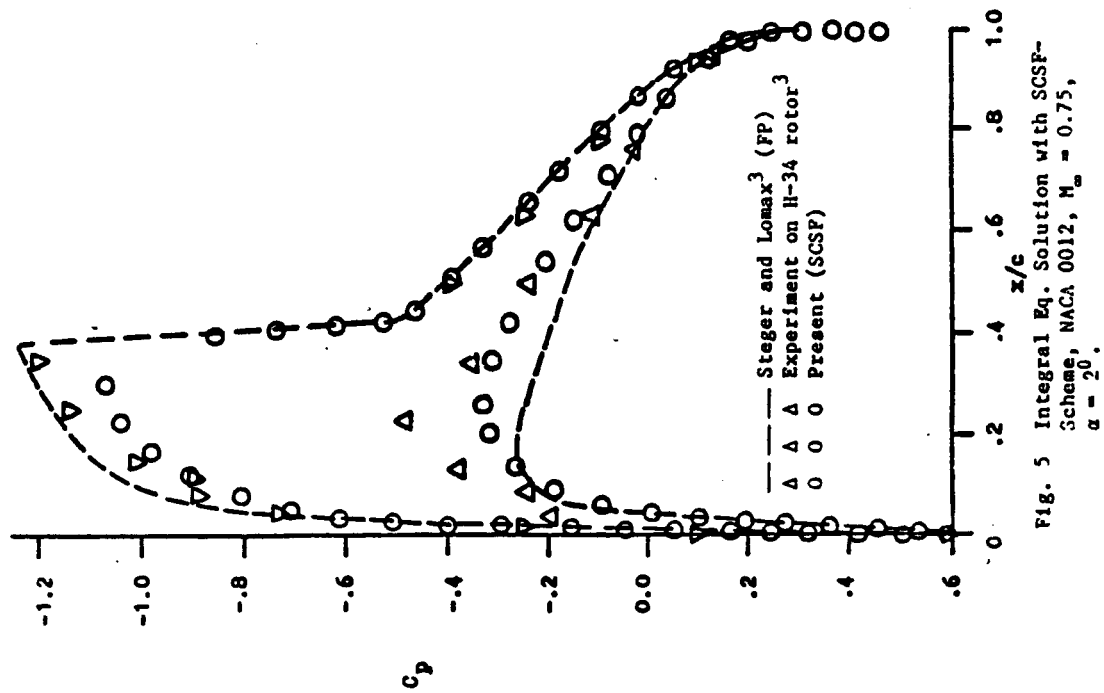


Fig. 5 Integral Eq. Solution with SCSF-Scheme, NACA 0012, $M_\infty = 0.75$, $\alpha = 20^\circ$.

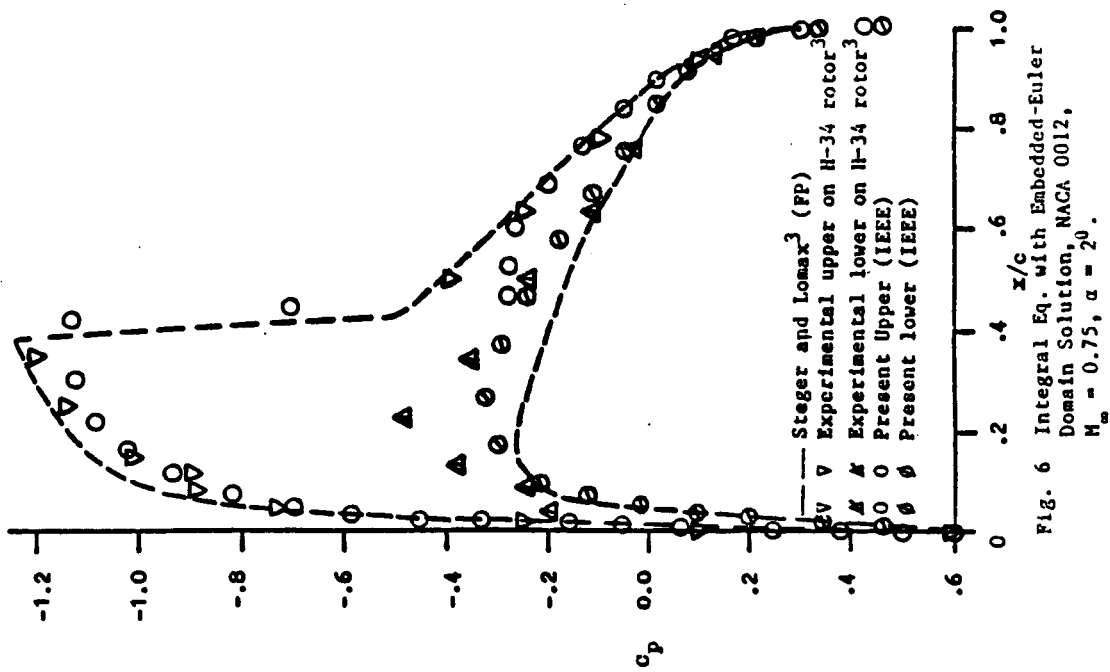


Fig. 6 Integral Eq. with Embedded-Euler Domain Solution, NACA 0012, $M_\infty = 0.75$, $\alpha = 20^\circ$.

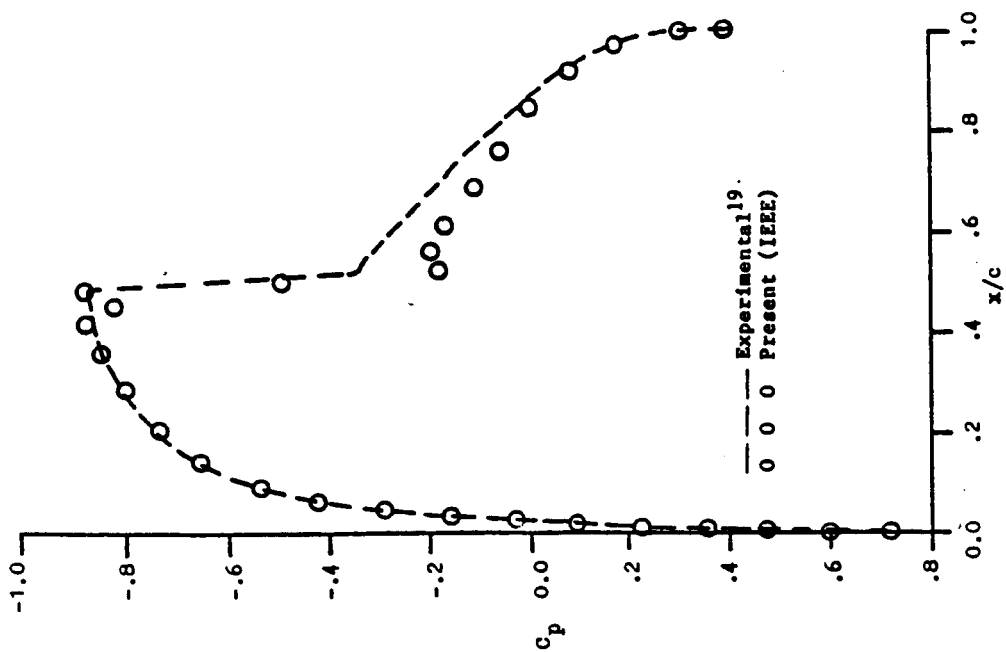


Fig. 7 Integral Eq. with Embedded-Euler Domain Solution, NACA 0012, $M_\infty = 0.812$, $\alpha = 0^\circ$.

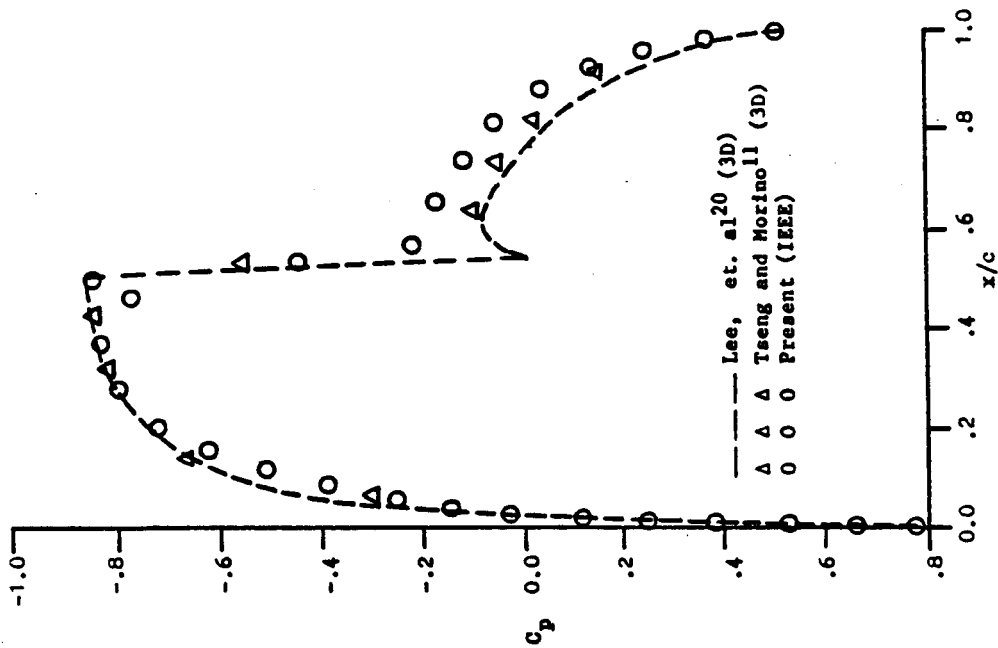


Fig. 8 Integral Eq. with Embedded-Euler Domain Solution, NACA 0012, $M_\infty = 0.82$, $\alpha = 0^\circ$.

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APPLICATIONS OF CLASSICAL AND ZERO-TOTAL-PRESSURE-LOSS

SETS OF EULER EQUATIONS TO DELTA WINGS

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OUTLINE OF THE TALK

1. MOTIVATION AND OBJECTIVES
2. FORMULATION
 - CLASSICAL AND ZERO-TOTAL-PRESSURE-LOSS SETS
 - SUPERSONIC CONICAL FLOW EQUATIONS
 - RELATIVE MOTION IN A ROTATING FRAME OF REFERENCE
3. HIGHLIGHTS OF METHOD OF SOLUTION
4. APPLICATIONS:
 - CONICAL FLOW, SHARP-EDGED WINGS (CLASSICAL AND ZTPL SETS)
 - CONICAL FLOW, ROUND-EDGED WINGS (CLASSICAL AND ZTPL SETS)
 - THREE-DIMENSIONAL FLOWS; TRANSONIC AND LOW-SPEED FLOWS
 - UNIFORM ROLLING IN A CONICAL FLOW
 - ROLLING OSCILLATION IN A LOCALLY CONICAL FLOW

Numerical
Examples (1)

Numerical
Examples (2)
5. CONCLUDING REMARKS

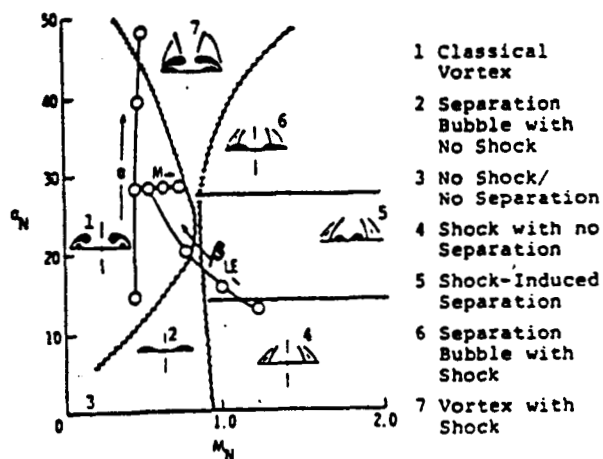


Figure 1. Miller and Wood¹ Classification Diagram.

CLASSICAL EULER EQUATIONS

• CONSERVATION FORM OF EULER EQUATIONS IN A SPACE-FIXED FRAME OF REFERENCE

$$\frac{\partial \bar{q}}{\partial t} + \frac{\partial \bar{E}}{\partial x} + \frac{\partial \bar{F}}{\partial y} + \frac{\partial \bar{G}}{\partial z} = 0 \quad (1)$$

$$\bar{q} = [\rho, \rho u, \rho v, \rho w, \rho e]^t \quad (2)$$

$$\bar{E} = [\rho u, \rho u^2 + p, \rho uv, \rho uw, \rho uh]^t \quad (3)$$

$$\bar{F} = [\rho v, \rho uv, \rho v^2 + p, \rho vw, \rho vh]^t \quad (4)$$

$$\bar{G} = [\rho w, \rho uw, \rho vw, \rho w^2 + p, \rho wh]^t \quad (5)$$

$$e = p/\rho(\gamma-1) + (u^2 + v^2 + w^2)/2 \quad (6)$$

$$h = e + p/\rho \quad (7)$$

SUPERSONIC CONICAL FLOW EQUATIONS

CONICAL VARIABLES

$$\xi = x, \eta = y/x, \zeta = z/x \quad (6)$$

CONICAL FLOW EQUATIONS

$$\frac{\partial \bar{q}}{\partial t} + \frac{\partial \tilde{F}}{\partial \eta} + \frac{\partial \tilde{G}}{\partial \zeta} + 2 \bar{E} = 0 \quad (7)$$

WHERE

$$\tilde{F} = \bar{F} - \eta \bar{E} \quad (8)$$

$$\tilde{G} = \bar{G} - \zeta \bar{E} \quad (9)$$

Zero-Total-Pressure-Loss Euler Equations

- Replace the energy equation by either one of the isentropic gas equations

$$p/\rho^\gamma = \text{const. or } \frac{\partial}{\partial t}(\rho s) + \nabla \cdot (\rho s \bar{v}) = 0$$

- Set (1):
- Replace the x-momentum equation (second elements in the vectors given in Eqs. (2) and (3)) by the steady energy equation (total constant enthalpy)

$$h = \text{const} = \frac{\gamma p}{(\gamma - 1)\rho} + \frac{1}{2} (u^2 + v^2 + w^2)$$

- Set (2):
- Replace the continuity equation (first elements in the vectors given in Eqs. (2) and (3)) by the steady energy equation given in Set (1)

EXPLANATION OF TOTAL-PRESSURE CHANGE FOR CLASSICAL AND ZTPL SETS OF EULER EQUATIONS

DIFFERENTIAL EULER EQUATIONS

CROCCO'S THEOREM

$$T \nabla \cdot S = \bar{\omega} \times \bar{v} + \frac{\partial \bar{v}}{\partial t} + \nabla h \quad (1)$$

DEFINITION OF ENTROPY CHANGE

$$\Delta S = R \ln \frac{P_{T_0}}{P_T} + C_p \ln \frac{T_0}{T} \quad (2)$$

(A) CLASSICAL SET

STEADY FLOW $\frac{\partial \bar{v}}{\partial t} = 0$, $h = \text{const}$ AND

$$T \nabla \cdot S = \bar{\omega} \times \bar{v}, \quad \Delta S = R \ln \frac{P_{T_0}}{P_T}$$

FOR A FREE-SHEET $\bar{\omega}$ IS PARALLEL TO \bar{v} ! $\nabla \cdot S = 0 \rightarrow P_{T_0} = P_T + \text{ZERO-TOTAL-PRESSURE LOSS}$

(B) ZERO-TOTAL-PRESSURE-LOSS SET (SHOCK-FREE AND WEAK SHOCKS)

$$h = \text{const}, \quad \nabla \cdot S = 0$$

AND $\bar{\omega}$ MUST BE PARALLEL TO \bar{v} , $P_{T_0} = P_T + \text{ZERO-TOTAL-PRESSURE LOSS}$

COMPUTATIONAL EULER EQUATIONS

CROCCO'S THEOREM

$$T \Delta S = \bar{\omega} \times \bar{V} + \frac{\partial \bar{V}}{\partial t} + \nabla h - \frac{1}{\rho} \nabla \cdot \bar{\tau} \quad \text{VISCOUS-FORM OF THE EQUATION} \quad (1)$$

DEFINITION OF ENTROPY CHANGE

$$\Delta S = R \ln \frac{P_{T_{\infty}}}{P_T} + C_p \ln \frac{T_0}{T_{0_{\infty}}} \quad (2)$$

(A) CLASSICAL SET

FOR STEADY FLOW $\frac{\partial \bar{V}}{\partial t} = 0$, $h = \text{const}$ AND

$$T \nabla S = \bar{\omega} \times \bar{V} + \text{Numerical Dissipation}, \Delta S = R \ln \frac{P_{T_{\infty}}}{P_T}$$

EVEN IF $\bar{\omega}$ IS PARALLEL TO \bar{V} , $\nabla S \neq 0 \rightarrow P_{T_{\infty}} \neq P_T \rightarrow \text{NON-ZERO TPL}$

(B) ZERO-TOTAL-PRESSURE-LOSS SET (SHOCK-FREE AND WEAK SHOCKS)

$h = \text{const}$, $\Delta S = 0$ AND

$$0 = \bar{\omega} \times \bar{V} + \text{Numerical Dissipation}, P_{T_{\infty}} = P_T \rightarrow \text{ZERO TPL}$$

CLASSICAL EULER EQUATIONS FOR THE RELATIVE MOTION IN A ROTATING FRAME OF REFERENCE

- THE CONSERVATION FORM OF THE CLASSICAL EULER EQUATIONS FOR THE ABSOLUTE MOTION OF THE FLOW IN A SPACE-FIXED FRAME OF REFERENCE

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{V}) = 0 \quad (1)$$

$$\frac{\partial (\rho \bar{V})}{\partial t} + \nabla \cdot (\rho \bar{V} \bar{V} + p \bar{I}) = 0 \quad (2)$$

$$\frac{\partial (\rho e)}{\partial t} + \nabla \cdot (\rho h \bar{V}) = 0 \quad (3)$$

$$e = p/\rho(\gamma-1) + \frac{V^2}{2} \quad (4)$$

$$h = e + p/\rho \quad (5)$$

- To express these equations in terms of a moving frame of reference, we use the following relations of the substantial and local derivatives for a scalar "a" and a vector "A":

$$\frac{Da}{Dt} = \frac{D'a}{Dt'} \quad (6.a)$$

$$\frac{\partial a}{\partial t} = \frac{\partial' a}{\partial t'} - \bar{V}_t \cdot \nabla a \quad (6.b)$$

$$\frac{D\bar{A}}{Dt} = \frac{D'\bar{A}}{Dt'} + \bar{\omega} \times \bar{A} \quad (7.a)$$

$$\frac{\partial \bar{A}}{\partial t} = \frac{\partial' \bar{A}}{\partial t'} - \bar{V}_t \cdot \nabla \bar{A} + \bar{\omega} \times \bar{A} \quad (7.b)$$

- The transformation velocity \bar{V}_t is a function of the moving frame of reference translation and rotation

$$\bar{V}_t = \bar{V} - \bar{V}_r = \bar{V}_0 + \bar{\omega} \times \bar{r} \quad (8)$$

- Restricting the motion of the frame of reference to the rotational motion,

$$\bar{V}_0 = 0 \text{ and } \frac{D\bar{V}_0}{Dt} = 0,$$

- the equations of relative motion in the rotating frame of reference

$$\frac{\partial' \rho}{\partial t'} + \nabla \cdot (\rho \bar{V}_r) = 0 \quad (9)$$

$$\frac{\partial' (\rho \bar{V}_r)}{\partial t'} + \nabla \cdot [\rho \bar{V}_r \bar{V}_r + \rho \bar{I}] = -\rho [\dot{\bar{\omega}} \times \bar{r} + 2\bar{\omega} \times \bar{V}_r + \bar{\omega} \times (\bar{\omega} \times \bar{r})] \quad (10)$$

$$\frac{\partial' (\rho e_r)}{\partial t'} + \nabla \cdot [\rho h_r \bar{V}_r] = -\rho [\bar{V}_r \cdot (\dot{\bar{\omega}} \times \bar{r}) + (\bar{\omega} \times \bar{r}) \cdot (\dot{\bar{\omega}} \times \bar{r})] \quad (11)$$

where

$$e_r = \frac{\rho}{\rho(\gamma-1)} + \frac{V_r^2}{2} - \frac{1}{2} |\bar{\omega} \times \bar{r}|^2 = e - \bar{V} \cdot (\bar{\omega} \times \bar{r}) \quad (12)$$

$$h_r = \frac{\gamma p}{\rho(\gamma-1)} + \frac{V_r^2}{2} - \frac{1}{2} |\bar{\omega} \times \bar{r}|^2 = h - \bar{V} \cdot (\bar{\omega} \times \bar{r}) \quad (13)$$

- The abstract conservative form of the relative motion in terms of the rotating coordinates (x', y', z') is given by

$$\frac{\partial' \bar{q}_r}{\partial t'} + \frac{\partial' \bar{E}_r}{\partial x'} + \frac{\partial' \bar{F}_r}{\partial y'} + \frac{\partial' \bar{G}_r}{\partial z'} = \bar{S} \quad (14)$$

where

$$\bar{q}_r = [\rho, \rho u_r, \rho v_r, \rho w_r, \rho e_r]^t \quad (15)$$

$$\bar{E}_r = [\rho u_r, \rho u_r^2 + p, \rho u_r v_r, \rho u_r w_r, \rho u_r h_r]^t \quad (16)$$

$$\bar{F}_r = [\rho v_r, \rho u_r v_r, \rho v_r^2 + p, \rho v_r w_r, \rho v_r h_r]^t \quad (17)$$

$$\bar{G}_r = [\rho w_r, \rho u_r w_r, \rho v_r w_r, \rho w_r^2 + p, \rho w_r h_r]^t \quad (18)$$

$$\bar{S} = [0, 0, \rho(\dot{\omega} z + 2\omega w_r + \omega^2 y), -\rho(\dot{\omega} y + 2\omega v_r - \omega^2 z), -\rho(-v_r \dot{\omega} z + w_r \dot{\omega} y + \omega \dot{\omega} y^2 + \omega \dot{\omega} z^2)]^t \quad (19)$$

- Since only the rolling motion is solved, the source term \bar{S} has been written for $\bar{\omega} = \omega \bar{e}_x$, and $\dot{\bar{\omega}} = \dot{\omega} \bar{e}_x$.

HIGHLIGHTS OF METHOD OF SOLUTION

1. WE USE THE CENTRAL-DIFFERENCE FINITE-VOLUME SCHEME WITH FOUR-STAGE RUNGE KUTTA TIME STEPPING AND EXPLICIT SECOND- AND FOURTH-ORDER DISSIPATION TERMS.
2. FOR STEADY FLOWS, LOCAL-TIME STEPPING IS USED, AND FOR UNSTEADY FLOWS MINIMUM GLOBAL TIME STEPPING IS USED.
3. A THREE-DIMENSIONAL COMPUTER PROGRAM IS USED TO SOLVE FOR:
 - CONICAL FLOWS (USING 3 CONICAL PLANES, WE ENFORCE THE ABSOLUTE FLOW VECTOR TO BE EQUAL ON THESE PLANES)
 - DIRECT SOLUTION OF THE THREE-DIMENSIONAL FLOW PROBLEM.
4. DEPENDING ON THE PROBLEM UNDER CONSIDERATION, DIFFERENT INITIAL CONDITIONS ARE USED.
5. DEPENDING ON THE PROBLEM UNDER CONSIDERATION, DIFFERENT SURFACE, FARFIELD AND SYMMETRY CONDITIONS ARE USED. FOR SUPersonic FLOWS, THE OUTER ROW SHOCK IS CAPTURED AS PART OF THE SOLUTION.

NUMERICAL EXAMPLES (1)

- SHARP-EDGED WINGS (CLASSICAL EULER EQS. & ZERO-TOTAL-PRESSURE-LOSS SETS)
- ROUND-EDGED WINGS (CLASSICAL EULER EQS. & ZERO-TOTAL-PRESSURE-LOSS SETS)
 - NUMERICAL BOUNDARY CONDITION (COARSE AND FINE GRIDS)
 - CLOSED FORM BOUNDARY CONDITION (COARSE AND FINE GRIDS)
- THREE-DIMENSIONAL TRANSONIC AND SUBSONIC FLOWS

Figure 1. Standard Euler Set, Sharp-edged Wing, 64X64 Cell,
 $M_\infty = 2.0$, $\alpha = 10^\circ$, $\beta = 70^\circ$, $\epsilon_2 = 0.12$, $\epsilon_4 = 0.005$, 1. Surface pressure, 2.
 Crossflow Mach number, 3. Crossflow Velocity.

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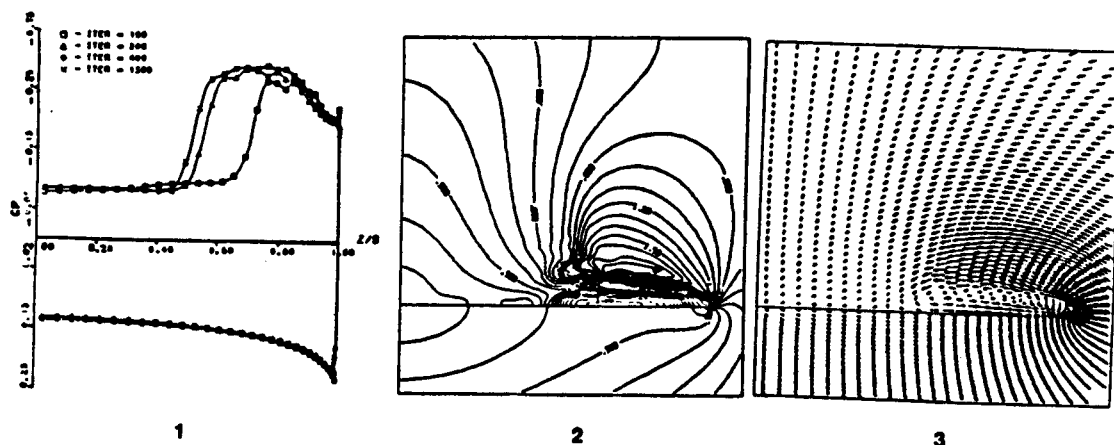


Figure 2. Zero-Total-Pressure-Loss Euler, Set (1), Sharp-edged Wing, 64x64
 Cell, $M_\infty = 2.0$, $\alpha = 10^\circ$, $\beta = 70^\circ$, $\epsilon_2 = 0.12$, $\epsilon_4 = 0.005$, 1. Surface Pressure,
 2. Crossflow Mach number, 3. Crossflow Velocity.

